Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R) (FT/WP) Examination November 2025 (2024 Scheme)

Course Code: GYMAT301

Course Name: MATHEMATICS FOR PHYSICAL SCIENCE – 3

Max. Marks: 60 Duration: 2 hours 30 minutes

PART A

(Answer all questions. Each question carries 3 marks)

		CO	Marks
1	State the conditions under which a function $f(x)$ has a Fourier integral representation and write the Fourier integral representation of the function under the conditions.	1	(3)
2	Find the Fourier sine transform of the function $f(x) = \begin{cases} 1; & if \ 0 < x < 1 \\ 0; & if \quad x > 1 \end{cases}$	1	(3)
3	Find $Re(f)$ and $Im(f)$ of the function $f(z)$ given by $f(z) = \frac{1}{1-z}$.	2	(3)
4	Sketch and shade the region in the complex plane give by $\lvert z - 1 + i \rvert \leq \sqrt{2}$.	2	(3)
5	Evaluate $\oint_C \frac{2}{z-4i}\, dz$ where C is the circle $\lvert z \rvert = \pi$ counter clockwise.	3	(3)
6	Evaluate $\int_C sec^2 z\, dz$ where C is any path from $z = -\pi/4$ to $z = \pi/4$.	3	(3)
7	Find the location and order of zeros of $f(z) = sin^2 z - 1$.	4	(3)
8	Find the residue of the pole $z = 0$ of the function $f(z) = z^{-5} \cos z$.	4	(3)

PART B

(Answer any one full question from each module, each question carries 9 marks)

Module -1

Q. No.		Question	CO	Marks
9	a)	Find the Fourier integral representation of the function $f(x)$ given by $f(x) = \begin{cases} 1, & if\ \lvert x\rvert < 2 \\ 0, & if\ \lvert x\rvert > 2 \end{cases}$. Find the function to which it converges.	1	5
	b)	If $f(x)$ is continuous and absolutely integrable on the x-axis, $f'(x)$ is piecewise continuous on every finite interval and if $f(x) \to 0$ as $x \to \infty$, then prove that (i) $\mathcal{F}_c[f'(x)] = \omega \mathcal{F}_s\,[f(x)] - \sqrt{\frac{2}{\pi}} f(0)$, (ii) $\mathcal{F}_s[f'(x)] = -\omega \mathcal{F}_c\,[f(x)]$.	1	4
10	a)	Find the Fourier transform of $f(x) = \begin{cases} 1-\lvert x\rvert & if\ \ \lvert x\rvert < 1 \\ 0 & Otherwise \end{cases}$. Hence evaluate $\int_0^\infty \frac{1-\cos x}{x^2}\,dx$.	1	5
	b)	Find the Fourier cosine integral of $f(x) = e^{-2x}, x > 0$. Hence evaluate $\int_0^\infty \frac{\cos \omega x}{4+\omega^2}\,dx$.	1	4

Module -2

Q. No.		Question	CO	Marks
11	a)	Check whether the function $u(x,y) = e^x \sin y$ is harmonic. If yes, find the harmonic conjugate $\mathrm{v}(x,y)$ and the analytic function $f(z) = u + iv$.	2	5
	b)	Find and sketch the image of the region $1 < \lvert z\rvert < 2,\ 0 < Arg\ z < \frac{\pi}{2}$ under the mapping $w = z^2$.	2	4
12	a)	Define an analytic function in a domain D. Show that $f(z) = \lvert z\rvert^2$ is differentiable only at $z = 0$ and hence it is nowhere analytic.	2	5
	b)	Determine the image of the straight line $x = 1$ under the mapping $w = \frac{1}{z}$. Sketch the image.	2	4

Module -3

Q. No.		Question	CO	Marks
13	a)	Find $\int_C z\, e^{z^2}\,dz$ where C is the path through the axes from $z = 1$ to $z = i$.	3	5
	b)	Evaluate $\int_C \frac{2z}{(z-1)(z-3)}\,dz$ where C is the circle given by: (i) $\lvert z-1\rvert = 1$, (ii) $\lvert z-2\rvert = 2$.	3	4
14	a)	Evaluate $\oint_C \frac{3z-1}{z^2-z-2}\,dz$ where C is the circle given by (i) $\lvert z-1-i\rvert = 3$.	3	5

	b)	Evaluate $\int_C \bar{z}\, dz$ where C is parametrized by $z(t) = 3t + it^2, -1 \leq t \leq 4$.	3	4

Module -4

15	a)	Evaluate $\int_0^{2\pi} \frac{1}{5-2\sin\theta}\, d\theta$.	4	5
	b)	Find the first three non-zero terms of the Taylor's series of the function $f(z) = cos^2 z$ at the point $z = \pi$.	4	4
16	a)	Find all the poles and nature of poles of the function $f(z) = \frac{z+1}{z^3-2z^2}$. By finding the residues at the pole, evaluate $\oint_C \frac{z+1}{z^3-2z^2}\, dz$ where C is the circle $\lvert z-1\rvert = 2$.	4	5
	b)	Find the Laurent's series representation of the function $f(z) = \frac{2z+3}{z^2+z-6}$ in the region $2 \leq \lvert z\rvert \leq 3$.	4	4

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